**Applied Computational Science**

# Homework: Linear Algebra.

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**Q.1 a)**

To show *ATA* symmetric,

Since transpose of the term ATA is gives the original matrix as result.

For Positive Semi-definite,

We need to show *≥ 0*,

Since square of any real number is always positive.

To demonstrate that *ATA* is positive definite,

Assume *A* is invertible,

*xT*(*ATA*)*x*=(*Ax*)*T*(*Ax*)=∥*Ax*∥2

And, we know, for . Therefore, which means is positive definite.

Also, let us assume that is positive definite, then for a non-zero vector ,

If A was invertible, then the value of should be , which is not possible since x is a non-zero vector, A must be invertible.

**Q.1 b)**

The given matrix is,

Transpose of will be,

Since, is symmetric, the term will be symmetric, and subtracting a symmetric matrix from the identity matrix results in a symmetric matrix. Therefore,

Since is orthogonal, its inverse will be equal to the transpose.

Therefore,

**Q.1 c)**

If is symmetric then,

Hence, will also be symmetric.

**Q2. a)**

Let be a strictly upper triangular matrix of size ,

The eigenvector of A will be,

Where, is non-zero.

**Q2. B)**

The equation relating , where is an orthogonal matrix, can be given by,

This method is much cheaper for computing than carrying out 99 matrix multiplications.

So to compute ,

**Q2. C)**

If is an upper triangular matrix of 1’s, to solve the linear system , in which is given and is unknown, we can find the values of by,

And so on …,

**Q.3 a)**

We have the matrix,

For ,

For ,

For , will be,

Which is equal to , hence holds true for

For ,

We know that,

Assuming the given relation is true,

We know the value of ,

Therefore, the given relation holds true for

**Q.3 b)**

Value of will be,

Since we know ,